Some Misconceptions in Derivative Pricing

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ABSTRACT

This paper has used the Arbitrage Theorem (Gordan Theorem) to clarify some misconceptions in the literature of derivative pricing. First, unlike the claim of the irrelevancy of the underlying asset's (stock's) expected return, it is found that the value of an option depends on the probability of the underlying asset (stock) rising or falling. Using the relationship between the relative price ratio between the two states: $\pi/(1-\pi)$ and the probability of the up move, the paper also derives discrete-time versions of the Greeks. Second, since with no arbitrage, μ is a function of r_f and σ, the Black-Scholes option pricing formula contains the underlying asset's expected rate of return μ. Third, with a two-step contract, it has been shown that within a company, there is no first claim or seniority between bond and stock, but there is first claim among fixed-income assets (e.g., labor and bond), and labor is senior to bond.

Key words: Arbitrage Theorem, irrelevancy of the underlying asset's expected return, discrete-time versions of the Greeks, first claim between bond and stock.

JEL Classification: G13, G32.

1. Introduction

The seminal work of Black and Scholes (1973) has inspired many researches on pricing and hedging different financial contracts. The literature argues that when pricing options, the value of an option does not depend on the probability of the underlying asset (stock) rising or falling (e.g., Avellaneda and Laurence, 1999; Hull, 2012; Cox, Ross and Rubinstein, 1979; Kolb and Overdahl, 2007; Shreve, 2004a; Wilmott, 2007; among others). Black and Scholes also claim that the Black-Scholes option pricing formula does not contain underlying asset's expected rate of return μ. The corporate finance literature argues that within a company, bond is senior to stock (or bond has first claim over stock), and stock is more risky than bond. I think these arguments are not correct. In this paper, I use the Arbitrage Theorem to show that first, the value of an option depends on the probability of the stock rising or falling. Using the relationship between the relative price ratio between the two states: $\pi/(1-\pi)$ and the probability of the up move, I also derive discrete-time versions of the Greeks. Second, since with no arbitrage, μ is a function of r_f and σ, the Black-Scholes option pricing formula contains underlying asset's expected rate of return μ. Third, with a two-step contract, it can be shown that there is no first claim or seniority between bond and stock, but there is first claim among fixed-income assets (e.g., labor and bond), and labor is senior to bond.

The remainder of this paper is organized as follows. Section 2 introduces the Arbitrage Theorem and uses the theorem and several examples to clarify some misconceptions in the literature. Concluding remarks appear in Section 3.

2. Arbitrage Theorem and Misconceptions in Derivative Pricing

Chang (2012) has introduced the Gordan Theorem (see also Bazaraa *et al.*, 1993, p.47).

Gordan Theorem (Arbitrage Theorem):
Let $\mathbf{A}$ be an $m \times n$ matrix. Then, exactly one of the following systems has a solution:

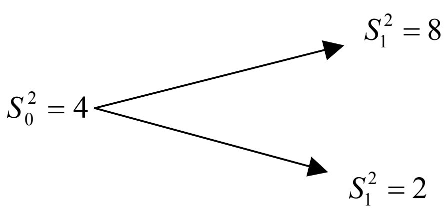
System 1: $\mathbf{Ax} > \mathbf{0}$ for some $\mathbf{x} \in R^n$

$$\text{System 2:} \quad \mathbf{A}^t\mathbf{p} = \mathbf{0} \quad \text{for some } \mathbf{p} \in R^m, \ \mathbf{p} \geq \mathbf{0}, \ \mathbf{e}^t\mathbf{p} = 1 \ \text{ where } \ \mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

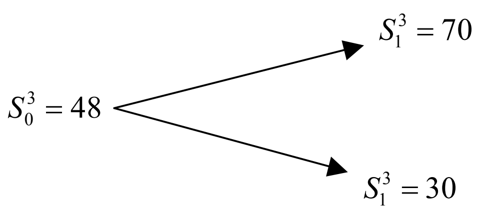
In System 2 of the Arbitrage Theorem, the vector $\mathbf{p}$ (which is not the same as the probability measure in the real world) is usually termed as the risk neutral probability measure, and p_i, $i = 1, ..., m$, can be interpreted as the current price of one dollar received at the end of period if state i occurs. If System 2 holds and the matrix $\mathbf{A}$ has rank m (i.e., the matrix has m independent rows), the risk neutral probability measure $\mathbf{p}$ will be unique.[1] If System 2 does not hold, as the following example shows, we can do arbitrage.

Example 1. Arbitrage Example.

Assume a one-period, two states (good time and bad time) of nature world with no transaction costs. There are a money market (Security 1) which provides $1 + 0.25$ dollars at time one if one dollar is invested at time 0 (i.e., the risk-free interest rate is $r = 0.25$), and two other securities (Security 2 and Security 3) with current prices 4 and 48 dollars, respectively, which provide:



Security 2



Security 3

Note that the two securities are not governed by the same risk neutral probability measure (i.e., System 2 of the Arbitrage Theorem has no solution):

[1] m independent rows of $\mathbf{A}$ means a complete market. See more discussions on incomplete and complete markets in

$$\begin{cases} \text{Security 2}: & S_0^2 = 4 = \dfrac{1}{1+0.25}\left(\dfrac{1}{2}\times 8 + \dfrac{1}{2}\times 2\right); \quad \mathbf{p'} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \\ \text{Security 3}: & S_0^3 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right); \quad \mathbf{p''} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix} \end{cases}$$

i.e., we cannot find a vector $\mathbf{p} = \begin{bmatrix} \pi \\ 1-\pi \end{bmatrix}$, $0 \le \pi \le 1$, such that System 2 holds:

$$\begin{bmatrix} 8-4(1+0.25) & 2-4(1+0.25) \\ 70-48(1+0.25) & 30-48(1+0.25) \end{bmatrix} \begin{bmatrix} \pi \\ 1-\pi \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

By System 1 of the Arbitrage Theorem, there must exist arbitrage opportunity. For example, at time 0, we can short sell one share of Security 3 and buy 5 shares of Security 2 and invest $28\,(=48-4\times 5)$ dollars in the money market, and at time 1 we can obtain net profit:

$$\begin{bmatrix} 8-4(1+0.25) & 70-48(1+0.25) \\ 2-4(1+0.25) & 30-48(1+0.25) \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, in equilibrium (with no arbitrage), the time-0 prices of Security 2 and Security 3 will change so that they can be priced by the same risk neutral probability measure, say, $\mathbf{p} = \begin{bmatrix} \pi \\ 1-\pi \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$, and

$$\begin{cases} \text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\ \text{Security 2}: & S_0^2 = 5.2 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 8 + \dfrac{1}{4}\times 2\right) \\ \text{Security 3}: & S_0^3 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \end{cases} \tag{1}$$

In this example, $\dfrac{\pi}{1-\pi} = \dfrac{3}{1}$ can be termed as the relative price ratio between the two states. That is, at time 1, the value of one dollar of good time is three times than that of bad time. We now use the

Arbitrage Theorem to clarify several misconceptions in the literature.

2.1. We Do Not Use Probabilities in Pricing Options?

Example 2. Binomial Option Pricing.

Assume a one-period, two states (good time and bad time) of nature world with no transaction costs. There are a money market with risk-free interest rate $r = 0.25$, an European call option C with strike price $K = 60$ dollars, and a stock with current price 48 dollars, which provide:

$$f_u = Max[S_0 \cdot u - K, 0] = Max[70 - 60, 0]$$

$$S_1 = S_0 \cdot u = 48(\frac{70}{48}) = 70$$

$$S_0 = 48$$

$$C = ?$$

$$S_1 = S_0 \cdot d = 48(\frac{30}{48}) = 30$$

$$f_d = Max[S_0 \cdot d - K] = Max[30 - 60, 0]$$

At time 0, by buying n shares of the underlying asset and selling one call to construct a portfolio which gives a certain time-1 payoff, the price of the European call is:[2]

$$\begin{cases} 70(n) - 10 = 30(n) - 0 \Rightarrow n = 0.25 \\ \dfrac{30(0.25) - 0}{1 + 0.25} = 48(0.25) - C \Rightarrow C = 6 \end{cases} \tag{2}$$

and by System 2 of the Arbitrage Theorem:

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[2] Chang (2012) has shown that in a complete market, all securities can be treated as a call option for each other. For example, in equation (1), Security 2 can be written as a call option for Security 3, where the strike price is: $K = 62$ if $S_1^3 = 70$ and $K = 28$ if $S_1^3 = 30$, and $\begin{cases} 70(n) - 8 = 30(n) - 2 \Rightarrow n = 0.15 \\ \dfrac{30(0.15) - 2}{1 + 0.25} = 48(0.15) - S_0^2 \Rightarrow S_0^2 = 5.2 \end{cases}$.

$$S_0 = 48 = \frac{1}{1+0.25}[\pi \times 70 + (1-\pi) \times 25] \Rightarrow \pi = \frac{3}{4} \text{ and } 1-\pi = \frac{1}{4} \quad \text{, and}$$

$$\begin{cases} \text{Money Market:} & 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 1.25 + \dfrac{1}{4} \times 1.25\right) \\ \text{Stock:} & S_0 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 70 + \dfrac{1}{4} \times 30\right) \\ \text{Call Option:} & C = 6 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 10 + \dfrac{1}{4} \times 0\right) \end{cases} . \tag{3}$$

It seems that the above option pricing formulas do not use probabilities. The finance literature also makes the same claim: e.g., "the option pricing formula does not involve the probabilities of the stock price moving up or down. For example, we get the same option price when the probability of an upward movement is 0.5 as we do when it is 0.9." (Hull, 2012, p. 257); "we do not need to know the probability that the stock will rise or fall" (Cox *et al.*, 1979, p. 232); "the probabilities of the up and down moves are irrelevant" (Shreve, 2004a, p.8); "the value of an option does not depend on the probability of the stock rising or falling. This is equivalent to saying that the stock growth rate is irrelevant for option pricing" (Wilmott, 2007, p. 65); among others.

I will argue that the claim of not using probabilities in valuing options is not correct. For example, assume that in Example 2, $S_0 d$ drops from 30 dollars to $S_0 d' = 25$ dollars, and other factors (i.e., K, u, S_0 and r) still remain the same:

Example 3. Binomial Option Pricing with Lower d'.

$$f_u = Max[S_0 \cdot u - K, 0] = Max[70 - 60, 0]$$

$$S_1 = S_0 \cdot u = 48(\frac{70}{48}) = 70$$

$$S_0 = 48$$

$$C' = ?$$

$$S_1 = S_0 \cdot d' = 48(\frac{25}{48}) = 25$$

$$f_d = Max[S_0 \cdot d' - K] = Max[25 - 60, 0]$$

and

$$\begin{cases} 70(n) - 10 = 25(n) - 0 \Rightarrow n = 2/9 \\ \dfrac{25(2/9) - 0}{1 + 0.25} = 48(2/9) - C' \Rightarrow C' = 6\dfrac{2}{9} \end{cases} . \quad (4)$$

Note that although the European call option's time-1 possible payoffs: $f_u = Max[70 - 60, 0] = 10$ and $f_d = Max[25 - 60, 0] = 0$ are the same as in Example 2, the call option price increases from $C = 6$ to $C' = 6\frac{2}{9}$. This is because, when the up move u and the interest rate r remain the same, and the down move d decreases, the current stock price will remain the same (i.e., $S_0 = 48$) only when investors (the market) believe the probability of the up move is higher than that in the previous case.[3] Once people believe that the up move of the underlying asset (the stock price) has higher probability, i.e., people believe $f_u = Max[70 - 60, 0] = 10$ has higher probability, the call option price will increase. Also, from System 2 of the Arbitrage Theorem:

$$S_0 = 48 = \frac{1}{1 + 0.25}[\pi' \times 70 + (1 - \pi') \times 25] \Rightarrow \pi' = \frac{7}{9} \text{ and } 1 - \pi' = \frac{2}{9} \text{ , and}$$

$$\begin{cases} \text{Money Market:} & 1 = \dfrac{1}{1 + 0.25}\left(\dfrac{7}{9} \times 1.25 + \dfrac{2}{9} \times 1.25\right) \\ \text{Stock:} & S_0 = 48 = \dfrac{1}{1 + 0.25}\left(\dfrac{7}{9} \times 70 + \dfrac{2}{9} \times 25\right) \\ \text{Call Option:} & C' = 6\dfrac{2}{9} = \dfrac{1}{1 + 0.25}\left(\dfrac{7}{9} \times 10 + \dfrac{2}{9} \times 0\right) \end{cases} . \quad (5)$$

The relative price ratio of (5) is $\frac{\pi'}{1 - \pi'} = \frac{7/9}{2/9} = \frac{3.5}{1}$, which is greater than $\frac{\pi}{1 - \pi} = \frac{3/4}{1/4} = \frac{3}{1}$ of (3). That is, the value of one dollar of good time is 3.5 times (not 3 times) than that of bad time. Higher valuation for good time's one dollar means that investors (the market) assign higher probability to good time.[4] With the relationship between π and the probability of the up move, we can derive the following discrete-time versions of the Greeks.

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[3] Also, it is hard to believe that (in Example 2) if investors think that the probability of 70 dollars is 99% and the probability of 30 dollars is 1%, the current stock price could still be the same 48 dollars (i.e., it should be very close to 70 dollars).

[4] We may say that our expectations about the future will affect our current behavior (valuation).

2.1.1 Discrete-Time Versions of the Greeks

(a) $\dfrac{\Delta C}{\Delta d} < 0$ and $\dfrac{\Delta P}{\Delta u} > 0$.

In Examples 2 and 3, where $S_0 \cdot d < K < S_0 \cdot u$ and d decreases to d',

$$C = \frac{1}{1+r}[\pi(S_0 \cdot u - K)] \quad , \quad C' = \frac{1}{1+r}[\pi'(S_0 \cdot u - K)] \quad ,$$

$$S_0 = \frac{1}{1+r}[\pi(S_0 \cdot u) + (1-\pi)(S_0 \cdot d)] \Rightarrow \pi = \frac{(1+r)-d}{u-d} \text{ and } 1-\pi = \frac{u-(1+r)}{u-d} \quad .$$

Because $\dfrac{C'}{C} = \dfrac{\pi'}{\pi} > 1$, we obtain $\dfrac{C'-C}{d'-d} \equiv \dfrac{\Delta C}{\Delta d} < 0$. For the European put option with $S_0 \cdot d < K < S_0 \cdot u$, suppose u increases to u', and other factors (i.e., K, d, S_0 and r) remain constant. Then, $\pi' < \pi$ and

$$P = \frac{1}{1+r}[(1-\pi)(K - S_0 \cdot d)] \quad , \quad P' = \frac{1}{1+r}[(1-\pi')(K - S_0 \cdot d)] \quad ,$$

we obtain $\dfrac{P'}{P} = \dfrac{1-\pi'}{1-\pi} > 1$ and $\dfrac{P'-P}{u'-u} \equiv \dfrac{\Delta P}{\Delta u} > 0$.

(b) Delta: $\dfrac{\Delta C}{\Delta S_0} > 0$ and $\dfrac{\Delta P}{\Delta S_0} < 0$.

When S_0 increases to S' and other factors (i.e., $K, S_0 d = S'd', S_0 u = S'u',$ and r) remain constant, it will mean higher probability for the up move (and $\pi' > \pi$) and thus, higher call option price and lower put option price:

$$S_0 = \frac{1}{1+r}[\pi(S_0 \cdot u) + (1-\pi)(S_0 \cdot d)] \quad , \quad S' = \frac{1}{1+r}[\pi'(S_0 \cdot u) + (1-\pi')(S_0 \cdot d)]$$

$$S' - S_0 = \frac{1}{1+r}[(\pi'-\pi)(S_0 \cdot u) - (\pi'-\pi)(S_0 \cdot d)] = \frac{1}{1+r}[(\pi'-\pi)S_0(u-d)]$$

$$\text{or,} \quad \pi'-\pi = (S' - S_0)\frac{1+r}{S_0(u-d)} > 0 \text{ when } S' > S_0 \ .$$

With $C = \frac{1}{1+r}[\pi(S_0 \cdot u - K)]$ and $C' = \frac{1}{1+r}[\pi'(S_0 \cdot u - K)]$, we have:

$$\frac{C'}{C} = \frac{\pi'}{\pi} > 1 \text{ and } \frac{\Delta C}{\Delta S_0} = \frac{\frac{1}{1+r}[(\pi' - \pi)(S_0 \cdot u - K)]}{S' - S_0} > 0. \tag{6}$$

With $P = \frac{1}{1+r}[(1-\pi)(K - S_0 \cdot d)]$ and $P' = \frac{1}{1+r}[(1-\pi')(K - S_0 \cdot d)]$, we have:

$$\frac{P'}{P} = \frac{1-\pi'}{1-\pi} < 1 \text{ and } \frac{\Delta P}{\Delta S_0} = \frac{\frac{-1}{1+r}[(\pi' - \pi)(K - S_0 \cdot d)]}{S' - S_0} < 0 \tag{7}$$

We can also derive the "elasticity" of the call option price with respect to the stock price:

$$\frac{\Delta C / C}{\Delta S_0 / S_0} = \frac{(\pi' - \pi)/\pi}{(S' - S_0)/S_0} = \frac{(\pi' - \pi)/\pi}{\frac{1}{1+r}[(\pi' - \pi)(u - d)]} = \frac{1+r}{\pi(u-d)} = \frac{1+r}{1+r-d} > 1 \quad \text{if } 0 < d < 1. \tag{8}$$

Thus, the call option is at least as volatile as the underlying asset (stock price).[5] The elasticity of the call option price with respect to the stock price depends only on r and d, and are independent of $S_0, u,$ and K. Also, the elasticity of the put option price with respect to the stock price is:

$$\frac{\Delta P / P}{\Delta S_0 / S_0} = \frac{-(\pi' - \pi)/(1-\pi)}{\frac{1}{1+r}[(\pi' - \pi)(u - d)]} = \frac{-(1+r)}{u - (1+r)} < 0, \tag{9}$$

which depends only on r and u, and will be less than -1 (i.e., the put option is more volatile than the stock) if $1 + r < u < 2(1+r)$; equal to -1 (i.e., the put option has the same volatility as the stock) if

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[5] Unlike in the Black-Scholes option model where call options are always more volatile than the stock price, in binomial option models, call options can have the same volatility as the stock price, e.g., in Example 2 $\frac{\Delta C / C}{\Delta S_0 / S_0} = 1$ if $d = 0$.

$1+r<u=2(1+r)$; greater than -1 (i.e., the put option is less volatile than the stock) if $u>2(1+r)$.[6]

(c) Rho: $\frac{\Delta C}{\Delta r}>0$ and $\frac{\Delta P}{\Delta r}<0$.

When the risk-free interest rate r increases to r', for the put option price, there will be two negative effects: (1) the present value of the put option's future payoff will decrease; (2) when r increases and S_0 still remains constant, it will mean higher probability for the up move of the stock price (and $\pi'>\pi$), which is bad for the put option price. For the call option price, an increase in r will have two opposite effects: (1) the present value of the call option's future payoff will decrease; (2) when r increases and S_0 still remains constant, it will mean higher probability for the up move of the stock price (and $\pi'>\pi$), which is good for the call option price.

$$S_0=\frac{1}{1+r}[\pi(S_0\cdot u)+(1-\pi)(S_0\cdot d)] \quad , \qquad S_0=\frac{1}{1+r'}[\pi'(S_0\cdot u)+(1-\pi')(S_0\cdot d)]$$

$$\Rightarrow \pi'-\pi=\frac{r'-r}{u-d}>0 \text{ if } r'>r$$

With $C=\frac{1}{1+r}[\pi(S_0\cdot u-K)]$ and $C'=\frac{1}{1+r'}[\pi'(S_0\cdot u-K)]$, we have:

$$\frac{C'}{C}=\frac{\frac{1+r}{(1+r)-d}}{\frac{1+r'}{(1+r')-d}}>1 \text{ if } r'>r; \quad \frac{C'}{C}=\frac{\frac{1+r}{(1+r)-d}}{\frac{1+r'}{(1+r')-d}}<1 \text{ if } r'<r, \text{ and } \frac{\Delta C}{\Delta r}>0. \tag{10}$$

With $P=\frac{1}{1+r}[(1-\pi)(K-S_0\cdot d)]$ and $P'=\frac{1}{1+r'}[(1-\pi')(K-S_0\cdot d)]$, we have:

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[6] It is interesting to note that $\left|(\frac{\Delta P/P}{\Delta S_0/S_0})/(\frac{\Delta C/C}{\Delta S_0/S_0})\right|=\frac{\pi}{1-\pi}$, i.e., the put option price can be more volatile, equally volatile, or less volatile than the call option price if investors believe that the probability of the up move of the stock price is larger, equal or smaller than that of the down move of the stock price. See more discussions in Appendix.

$$\frac{P'}{P}=\frac{\dfrac{1+r}{u-(1+r)}}{\dfrac{1+r'}{u-(1+r')}}<1 \text{ if } r'>r\,;\quad \frac{P'}{P}=\frac{\dfrac{1+r}{u-(1+r)}}{\dfrac{1+r'}{u-(1+r')}}>1 \text{ if } r'<r\,, \text{ and } \frac{\Delta P}{\Delta r}<0\,. \tag{11}$$

(c) Vega: Signs of $\dfrac{\Delta C}{\Delta(range)}$ and $\dfrac{\Delta P}{\Delta(range)}$ are undetermined.

From the following three examples:

(I).

$$\begin{tikzcd}[row sep=small] & S_1 = S_0 \cdot u = 70 \\ S_0 = 48 \arrow[ur] \arrow[dr] & \\ K = 60 & S_1 = S_0 \cdot d = 30 \end{tikzcd}$$

$$\begin{cases} \text{Money Market}: & 1=\dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25+\dfrac{1}{4}\times 1.25\right) \\ \text{Stock}: & S_0=48=\dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70+\dfrac{1}{4}\times 30\right) \\ \text{Call Option}: & C=6=\dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 10+\dfrac{1}{4}\times 0\right) \\ \text{Put Option}: & P=6=\dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 0+\dfrac{1}{4}\times 30\right) \end{cases}$$

(II).

$$\begin{tikzcd}[row sep=small] & S_1 = S_0 \cdot u = 68 \\ S_0 = 48 \arrow[ur] \arrow[dr] & \\ K = 60 & S_1 = S_0 \cdot d = 2 \end{tikzcd}$$

$$\begin{cases} \text{Money Market}: & 1 = \dfrac{1}{1+0.25}\left(\dfrac{58}{66}\times 1.25 + \dfrac{8}{66}\times 1.25\right) \\ \text{Stock}: & S_0 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{58}{66}\times 68 + \dfrac{8}{66}\times 2\right) \\ \text{Call Option}: & C = 5.62424 = \dfrac{1}{1+0.25}\left(\dfrac{58}{66}\times 8 + \dfrac{8}{66}\times 0\right) \\ \text{Put Option}: & P = 5.62424 = \dfrac{1}{1+0.25}\left(\dfrac{58}{66}\times 0 + \dfrac{8}{66}\times 58\right) \end{cases}$$

(III).

$$\begin{tikzcd} & S_1 = S_0 \cdot u = 90 \\ S_0 = 48 \arrow[ur] \arrow[dr] & \\ K = 60 & S_1 = S_0 \cdot d = 40 \end{tikzcd}$$

$$\begin{cases} \text{Money Market}: & 1 = \dfrac{1}{1+0.25}\left(\dfrac{2}{5}\times 1.25 + \dfrac{3}{5}\times 1.25\right) \\ \text{Stock}: & S_0 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{2}{5}\times 90 + \dfrac{3}{5}\times 40\right) \\ \text{Call Option}: & C = 9.6 = \dfrac{1}{1+0.25}\left(\dfrac{2}{5}\times 30 + \dfrac{3}{5}\times 0\right) \\ \text{Put Option}: & P = 9.6 = \dfrac{1}{1+0.25}\left(\dfrac{2}{5}\times 0 + \dfrac{3}{5}\times 20\right) \end{cases}$$

we can find that the range of the stock in (II) is larger than that in (I), and (II)'s call and put prices are lower. The range of the stock in (III) is also larger than that in (I), but (III)'s call and put prices are higher.[7]

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[7] Some may claim that "a rise in the variability of the stock will decrease its market value" (e.g., Ross, Westerfield and Jaffe, 2010, p. 689). However, this claim is not correct. For example, in a complete market with:

2.2. Black-Scholes Model Does Not Contain Stock's Expected Rate of Return?

Denote the stock price at time $0 \le t_i \le T$ as $X(t_i)$ where $X(0) \equiv S_0$ and $0 = t_0 < t_1 < ... < t_{n-1} < t_n = T$. Let

$$X(T) = \frac{X(T)}{X(t_{n-1})} \cdot \frac{X(t_{n-1})}{X(t_{n-2})} \cdot \ ... \ \cdot \frac{X(t_2)}{X(t_1)} \cdot \frac{X(t_1)}{X(0)} \cdot X(0) \ ,$$

and $Y(t_n) = X(T) / X(t_{n-1})$, $Y(t_{n-i}) = X(t_{n-i}) / X(t_{n-i-1})$, $n > i \ge 1$,

$$X(T) = Y(t_n) \cdot Y(t_{n-1}) \cdot \ ... \ \cdot Y(t_1) \cdot X(0) \quad \text{or} \quad \ell n X(T) = \sum_{i=1}^{n} \ell n(Y(t_i)) + \ell n(X(0))$$

Suppose that $\ell n Y(t_i)$, $i \ge 1$, are independent and identically distributed. Then, with large *n*, according to the Central Limit Theorem, $\sum_{i=1}^{n} \ell n Y(t_i) \equiv Y(T)$ is normally distributed, i.e., $Y(T) \sim N(\mu T, \sigma^2 T)$, and $X(T) = e^{\sum_{i=1}^{n} \ell n(Y(t_i)) + \ell n(X(0))} = S_0 \cdot e^{Y(T)}$, where $\ell n(X(0)) \equiv \ell n S_0$. Thus,

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$$\begin{cases} \text{Money Market (Security 1)}: & 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 1.25 + \dfrac{1}{4} \times 1.25\right) \\ \text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 70 + \dfrac{1}{4} \times 30\right) \\ \text{Security 3}: & S_0^3 = 48.2 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 72 + \dfrac{1}{4} \times 25\right) \\ \text{Security 4}: & S_0^4 = 47.6 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 72 + \dfrac{1}{4} \times 22\right) \\ \text{Security 5}: & S_0^5 = 47.8 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 68 + \dfrac{1}{4} \times 35\right) \\ \text{Security 6}: & S_0^6 = 48.4 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4} \times 68 + \dfrac{1}{4} \times 38\right) \end{cases}$$

both Securities 3 and 4 are more volatile than Security 2, but their current market values could be more (48.2) or less (47.6) than Security 2's market value (48). Both Securities 5 and 6 are less volatile than Security 2, and their current market values could be more (48.4) or less (47.8) than Security 2's market value (48).

$\left\{X(t)=S_0\cdot e^{Y(t)}, T\geq t\geq 0\right\}$ is a geometric Brownian motion process. Also, for $0\leq u\leq s\leq T$,

$$\begin{aligned}E[X(T)|X(u),0\leq u\leq s] &= E[S_0\cdot e^{Y(T)}|X(u),0\leq u\leq s]\\ &= S_0\cdot E[e^{Y(T)}|X(u),0\leq u\leq s]\\ &= S_0\cdot E[e^{Y(T)-Y(s)+Y(s)}|X(u),0\leq u\leq s]\\ &= S_0\cdot e^{Y(s)}E[e^{Y(T)-Y(s)}|X(u),0\leq u\leq s]\\ &= X(s)\cdot E[e^{Y(T)-Y(s)}]\end{aligned}$$

Since $(Y(T)-Y(s))\sim N(\mu(T-s),\sigma^2(T-s))$, we have

$$E[X(T)|X(u),0\leq u\leq s]=X(s)\cdot e^{(T-s)\mu+\frac{1}{2}(T-s)\sigma^2}=X(s)\cdot e^{(T-s)(\mu+\frac{1}{2}\sigma^2)}\quad. \tag{12}$$

Let $s=0$, $E[X(T)|X(0)]=X(0)\cdot e^{T(\mu+\frac{1}{2}\sigma^2)}$. (13)

From System 2 of the Arbitrage Theorem, we have $E_P[e^{-r_fT}\cdot X(T)|X(0)]=X(0)$ or

$$E_P[X(T)|X(0)]=e^{r_fT}\cdot X(0) \tag{14}$$

where P is the risk-neutral probability measure, r_f is the annual risk-free interest rate, and $0<T\leq 1$ (e.g., for three months, $T=3/12$). With no arbitrage, (13) must be equal to (14), and hence,[8]

$$r_f=\mu+\frac{1}{2}\sigma^2 \tag{15}$$

The payoff of the European call option with exercise price K is:

[8] See also Ross (1993, p. 470).

$$(X(T)-K)^{+}=\begin{cases} X(T)-K & \text{if } X(T)\geq K \\ 0 & \text{if } X(T)<K \end{cases} .$$

With no arbitrage,

$$0=E_P[(X(T)-K)^{+}-Ce^{r_f T}\big|C] \tag{16}$$

where $X(T)=S_0\cdot e^{Y(T)}$, and $Y(T)\sim N(\mu T,\sigma^2 T)$. (16) can be rewritten as:

$$Ce^{r_f T}=E_P[(X(T)-K)^{+}]=\int_{-\infty}^{+\infty}(S_0\cdot e^{y}-K)^{+}\cdot\frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{\frac{-(y-\mu T)^2}{2\sigma^2 T}}dy \quad .$$

From $S_0e^{y}-K\geq 0$ we have $y\geq \ell n(\frac{K}{S_0})$ and

$$Ce^{r_f T}=\int_{\ell n(\frac{K}{S_0})}^{+\infty}(S_0\cdot e^{y}-K)\cdot\frac{1}{\sqrt{2\pi\sigma^2 T}}\cdot e^{\frac{-(y-\mu T)^2}{2\sigma^2 T}}dy \quad .$$

Let $w=(y-\mu T)/\sigma\sqrt{T}$, we have $dy=\sigma\sqrt{T}dw$ and

$$Ce^{r_f T}=S_0\cdot e^{\mu T}\cdot\frac{1}{\sqrt{2\pi}}\int_{a}^{+\infty}e^{\sigma w\sqrt{T}}\cdot e^{\frac{-w^2}{2}}dw-K\cdot\frac{1}{\sqrt{2\pi}}\int_{a}^{+\infty}e^{\frac{-w^2}{2}}dw \tag{17}$$

where $a=[\ell n(\frac{K}{S_0})-\mu T]/\sigma\sqrt{T}$.

With
$$\frac{1}{\sqrt{2\pi}}\int_{a}^{+\infty}e^{\sigma w\sqrt{T}}\cdot e^{\frac{-w^2}{2}}dw=e^{\frac{T\sigma^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}\int_{a}^{+\infty}e^{\frac{-(w-\sigma\sqrt{T})^2}{2}}dw$$

$$=e^{T\sigma^2/2}\cdot\text{Prob}\left\{N(\sigma\sqrt{T},1)\geq a\right\}=e^{T\sigma^2/2}\cdot\text{Prob}\left\{N(0,1)\geq a-\sigma\sqrt{T}\right\}$$

$$=e^{T\sigma^2/2}\cdot\text{Prob}\left\{N(0,1)\leq \sigma\sqrt{T}-a\right\}=e^{T\sigma^2/2}\cdot\phi(\sigma\sqrt{T}-a)$$

(17) can be rewritten as:

$$Ce^{r_f T}=S_0\cdot e^{(\mu+\sigma^2/2)T}\cdot\phi(\sigma\sqrt{T}-a)-K\cdot\phi(-a)\,. \tag{18}$$

Substitute (15): $\mu+\frac{\sigma^2}{2}=r_f$ or $\mu=r_f-\frac{\sigma^2}{2}$ into (18),

$$C=S_0\cdot\phi(\sigma\sqrt{T}-a)-K\cdot e^{-r_f T}\cdot\phi(-a)=S_0\cdot\phi(d_1)-K\cdot e^{-r_f T}\cdot\phi(d_2) \tag{19}$$

where $$d_1 = \frac{ln(\frac{S_0}{K}) + (r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad , \quad d_2 = d_1 - \sigma\sqrt{T} \ .$$

Equation (15) indicates that with no arbitrage, μ is a function of r_f and σ although the Black-Scholes formula (19) doesn't show μ. This result refutes the claims in the literature that "the expected return on the stock does not appear in equation … The option value as a function of the stock price is independent of the expected return on the stock" (Black and Scholes, 1973, p. 644); "The option price does not depend on the expected return on the common stock" (Merton, 1990, p. 282); among others.

2.3. Bondholder Has 'First Claim' Over Stockholder?

The corporate finance literature claims that within a company, bond is senior to stock (or bond has first claim over stock), e.g., "stockholders do receive more earnings per dollar invested, but they also bear more risk, because they have given lenders first claim on the firm's assets and operating income" (Myers, 1984, p.94). The following example shows, however, that the claim of bond's being senior to stock is not correct.

Example 4. Seniority Between Bond and Stock?

Assume a one-period, two states (good time and bad time) of nature world with no transaction costs. There are a money market (Security 1) with risk-free interest rate $r = 0.25$, and two other securities, Security 2 and Security 3, where Security 3 is a portfolio of two shares of stock of a totally equity-financed firm:

$$
\begin{cases}
\text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\
\text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \\
\text{Security 3 (Firm)}: & \begin{cases} \text{Stock 1}: \ E_0^1 = 150 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times \dfrac{437.5}{2} + \dfrac{1}{4}\times \dfrac{187.5}{2}\right) \\ \text{Stock 2}: \ E_0^2 = 150 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times \dfrac{437.5}{2} + \dfrac{1}{4}\times \dfrac{187.5}{2}\right) \end{cases}
\end{cases}
. \tag{20}
$$

Suppose that the firm (Security 3) changes Stock 2 into a bond (i.e., the firm has 50% debt):

$$
\begin{cases}
\text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\
\text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \\
\text{Security 3 (Firm)}: & \begin{cases} \text{Stock}: \ E_0 = 150 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 250 + \dfrac{1}{4}\times 0\right) \\ \text{Bond}: \ D_0 = 150 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 187.5 + \dfrac{1}{4}\times 187.5\right) \end{cases}
\end{cases}
. \tag{21}
$$

We can think that there is a "two-step contract" (rather than "seniority") between the stockholder and the bondholder: First, split the firm's income equally between the stockholder and bondholder; and second, if the bondholder's 50% share is more than 187.5 dollars (the upper bound), she will give out any additional money to the stockholder; and if the bondholder's 50% share is less than 187.5 dollars, the stockholder will use her 50% share to compensate the bondholder until the stockholder's share becomes zero, or the bondholder gets 187.5 dollars. Also, comparing (20) with (21), we can find that stock and bond are like two parties in a forward contract, i.e., at time-1, the bondholder (with a short position) will sell a stock to the stockholder (with a long position) at the price 187.5 dollars, and the stockholder will obtain the stock which at time-1 may be worth 250 dollars or nothing. That is, the bondholder is willing to sacrifice the chance of obtaining more than 187.5 dollar at the good time (i.e., 437.5/2 of (20) > 187.5

of (21)) in order to avoid the possibility of obtaining less than 187.5 dollar at the bad time (i.e., 187.5/2 of (20) < 187.5 of (21)). The stockholder, on the other hand, is willing to take the chance (the risk) of getting less at the bad time (i.e., 0 of (21) < 187.5/2 of (20)) to gain the opportunity of obtaining more at the good time (i.e., 250 of (21) > 437.5/2 of (20)).[9]

Note that if bond has first claim over stock, bond should be less risky than stock. But, because bond's payment has an upper bound, bondholder will still get the same upper bound payment when things become much better and will obtain less when things become much worse. For example, in (21) if the future payment of Security 3 (the firm) becomes more volatile than expected (e.g., 600 dollars at the good time and 100 dollars at the bad time):

$$\begin{cases} \text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\ \text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \\ \text{Security 3 (Firm)}: & \begin{cases} \text{Stock}: E_0 = 247.5 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 412.5 + \dfrac{1}{4}\times 0\right) \\ \text{Bond}: D_0 = 132.5 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 187.5 + \dfrac{1}{4}\times 100\right) \end{cases} \end{cases}, \qquad (22)$$

the bondholder still gets the same 187.5 dollars at the good time but obtains much less (i.e., 100 < 187.5) at the bad time, and hence, the bond's current market value drops from 150 to 132.5 dollars. The stockholder, on the other hand, benefits (i.e., the stock's current market value increases from 150 to 247.5 dollars). This example shows that bond is more risky than stock. It also casts doubt on Knight's claim that within a company, "the confident and venturesome 'assume the risk' or 'insure' the doubtful and timid by guaranteeing to the latter a specified income in return for …power to direct his work" (p. 269).[10]

Example 5. Seniority Among Fixed-Income Assets.

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[9] Just like in the forward contract case where no party will compensate another party for bearing any kind of risk, bondholder will not compensate stockholder for so called first claim or seniority.

[10] Some practitioners also question the notion of bond's first claim and safety, e.g., "it is not the obligation that creates the safety, nor is it the legal remedies of the bondholder in the event of default. *Safety depends upon and is measured entirely by the ability of the debtor corporation to meet its obligations*" (Graham and Dodd, 1988, p. 113).

Assume a one-period, two states (good time and bad time) of nature world with no transaction costs. There are a money market (Security 1) with risk-free interest rate $r = 0.25$, and two other securities, Security 2 and Security 3 (firm), where Security 3 (firm) is a portfolio of a stock and two fixed-income assets: a labor and a bond,

$$\begin{cases} \text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\ \text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \\ \text{Security 3 (Firm)}: & \begin{cases} \text{Labor}: \; L_0 = 60 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 75 + \dfrac{1}{4}\times 75\right) \\ \text{Stock}: \; E_0 = 100 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 166\dfrac{2}{3} + \dfrac{1}{4}\times 0\right) \\ \text{Bond}: \; D_0 = 200 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 270\dfrac{5}{6} + \dfrac{1}{4}\times 187.50\right) \end{cases} \end{cases} . \tag{23}$$

It usually says that de jure, labor is senior to bond, and bond is senior to stock. But consider a two-step contract between the stock and the portfolio of labor and bond: First, split the firm's income between the stock ($27\frac{7}{9}\% = \frac{100}{360}$) and the portfolio of labor and bond ($72\frac{2}{9}\% = \frac{60+200}{360}$); and second, if the labor and bond portfolio's $72\frac{2}{9}\%$ share is more than the upper bound: $325\frac{5}{6} (= 75 + 270\frac{5}{6})$ dollars, the labor and bond portfolio will give out any additional money to the stockholder; and if the labor and bond portfolio's $72\frac{2}{9}\%$ share is less than $325\frac{5}{6}$ dollars, the stockholder will use her $27\frac{7}{9}\%$ share to compensate until the stockholder's share becomes zero, or the labor and bond portfolio gets $325\frac{5}{6}$ dollars. That is, there is no seniority or first claim between the stock and the portfolio of labor and the bond. However, since the two fixed-income assets' (labor's and the bond's) payments have upper bounds, when the bond and the labor split their income, the bondholder will have a less chance to obtain the upper bound payment $270\frac{5}{6}$ dollars (i.e., labor is senior to bond and bond is more risky than labor). From this example, we can conclude that within a company, there is no first claim or seniority between fixed-income assets (labor and bond) and non-fixed-income asset (stock), but there is first claim among fixed-income assets and labor is senior to bond.

3. Concluding Remarks

This paper has used the Arbitrage Theorem (Gordan Theorem) to clarify some misconceptions in the literature of derivative pricing. First, unlike the claim of the irrelevancy of the underlying asset's (stock's) expected return, it is found that the value of an option depends on the probability of the underlying asset (stock) rising or falling. Using the relationship between the relative price ratio between the two states: $\pi/(1-\pi)$ and the probability of the up move, the paper also derives discrete-time versions of the Greeks. Second, since with no arbitrage, μ is a function of r_f and σ, the Black-Scholes option pricing formula contains underlying asset's expected rate of return μ. Third, with a two-step contract, it has been shown that there is no first claim or seniority between bond and stock, but there is first claim among fixed-income assets (e.g., labor and bond), and labor is senior to bond.

Appendix

Assume a one-period, two states (good time and bad time) of nature world with no transaction costs. There are a money market with risk-free interest rate $r = 0.25$ and three securities with current market prices 48, 80 and 50 dollars, respectively:

$$\begin{cases} \text{Money Market (Security 1)}: & S_0^1 = 1 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 1.25 + \dfrac{1}{4}\times 1.25\right) \\ \text{Security 2}: & S_0^2 = 48 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 70 + \dfrac{1}{4}\times 30\right) \\ \text{Security 3}: & S_0^3 = 80 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times \dfrac{350}{3} + \dfrac{1}{4}\times 50\right) \\ \text{Security 4}: & S_0^4 = 50 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 75 + \dfrac{1}{4}\times 25\right) \\ \text{Call Option}: & C = 6 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 10 + \dfrac{1}{4}\times 0\right) \\ \text{Put Option}: & P = 6 = \dfrac{1}{1+0.25}\left(\dfrac{3}{4}\times 0 + \dfrac{1}{4}\times 30\right) \end{cases} \qquad \text{(A1)}$$

Note that C can be a call option for Security 2 (where $K = 60$), Security 3 (where $K = 320/3$), or Security 4 (where $K = 65$). Also, P can be a put option for Security 2 (where $K = 60$), Security 3 (where $K = 80$), or Security 4 (where $K = 55$). This result refutes Cox, Ross and Rubinstein's (1979) claim that "the only random variable on which the call value depends is the stock itself. In particular, it does not depend on the random prices of other securities or portfolios" (p. 235).

Suppose that $S_0^i, i = 2,3,4,$ increase, and other factors (i.e., $K, S_0 d = S'd', S_0 u = S'u', \text{and } r$) remain constant. This means that the probability of good time increases (and $\pi = \dfrac{3}{4}$ becomes $\pi' = \dfrac{13}{16}$):

$$\begin{cases} \text{Money Market (Security 1):} & S_0^{1'} = 1 = \frac{1}{1+0.25}\left(\frac{13}{16}\times 1.25 + \frac{3}{16}\times 1.25\right) \\ \text{Security 2:} & S_0^{2'} = 50 = \frac{1}{1+0.25}\left(\frac{13}{16}\times 70 + \frac{3}{16}\times 30\right) \\ \text{Security 3:} & S_0^{3'} = 83\frac{1}{3} = \frac{1}{1+0.25}\left(\frac{13}{16}\times \frac{350}{3} + \frac{3}{16}\times 50\right) \\ \text{Security 4:} & S_0^{4'} = 52.5 = \frac{1}{1+0.25}\left(\frac{13}{16}\times 75 + \frac{3}{16}\times 25\right) \\ \text{Call Option:} & C' = 6.5 = \frac{1}{1+0.25}\left(\frac{13}{16}\times 10 + \frac{3}{16}\times 0\right) \\ \text{Put Option:} & P' = 4.5 = \frac{1}{1+0.25}\left(\frac{13}{16}\times 0 + \frac{3}{16}\times 30\right) \end{cases} . \tag{A2}$$

The elasticity of the call option price with respect to the stock price is:

$$\frac{\Delta C/C}{\Delta S_0^2/S_0^2} = \frac{(6.5-6)/6}{(50-48)/48} = \frac{1+0.25}{1+0.25-(30/48)} = 2 = \frac{\Delta C/C}{\Delta S_0^3/S_0^3} = \frac{(6.5-6)/6}{(83\frac{1}{3}-80)/80} = \frac{1+0.25}{1+0.25-(50/80)},$$

$$\frac{\Delta C/C}{\Delta S_0^4/S_0^4} = \frac{(6.5-6)/6}{(52.5-50)/50} = \frac{1+0.25}{1+0.25-(25/50)} = \frac{5}{3},$$

the elasticity of the put option price with respect to the stock price is:

$$\frac{\Delta P/P}{\Delta S_0^2/S_0^2} = \frac{(4.5-6)/6}{(50-48)/48} = \frac{-(1+0.25)}{\frac{70}{48}-(1+0.25)} = -6 = \frac{\Delta P/P}{\Delta S_0^3/S_0^3} = \frac{(4.5-6)/6}{(83\frac{1}{3}-80)/80} = \frac{-(1+0.25)}{\frac{350/3}{80}-(1+0.25)},$$

$$\frac{\Delta P/P}{\Delta S_0^4/S_0^4} = \frac{(4.5-6)/6}{(52.5-50)/50} = \frac{-(1+0.25)}{\frac{75}{50}-(1+0.25)} = -5.$$

and $\left|(\frac{\Delta P/P}{\Delta S_0^i/S_0^i})/(\frac{\Delta C/C}{\Delta S_0^i/S_0^i})\right| = \frac{\pi}{1-\pi} = 3,\ i = 2,3,4.$

REFERENCES

Avellaneda, Macro and Peter Laurence, 1999, *Quantitative Modeling of Derivative Securities: From Theory To Practice*, Chapman and Hall/CRC, New York.

Baxter, Martin and Andrew Rennie, 1996, *Financial Calculus: An Introduction to Derivative Pricing*, Cambridge University Press., New York.

Bazaraa, Mokhtar, Hanif Sherali and C. M. Shetty, 1993, *Nonlinear Programming: Theory and Algorithms*, John Wiley & Sons, Inc., New York.

Black, Fischer and Myron Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637-654.

Chang, Kuo-Ping, 2012, "Are Securities Also Derivatives?," *American Journal of Operations Research*, 2: 430-441. doi: 10.4236/ajor.2012.23051. Also in http://ssrn.com/abstract=987522.

Cox, John, Stephen Ross, and Mark Rubinstein, 1979, "Option Pricing: A Simplified Approach," *Journal of Financial Economics* 7, 229-263.

Graham, Benjamin and David Dodd, 1988, *Security Analysis*, 6th edition, McGraw-Hill, New York.

Hull, John, 2012, *Options, Futures, and Other Derivatives*, Prentice Hall, New York.

Knight, Frank, 1933, *Risk, Uncertainty and Profit*, reprinted by the University of Chicago Press, Chicago, 1971.

Kolb, Robert and James Overdahl, 2007, *Futures, Options, and Swaps*, Blackwell, Malden, MA.

Merton, Robert, 1990, *Continuous-Time Finance*, Basil Blackwell, Cambridge, MA.

Myers, S. C., 1984, "The Search for Optimal Capital Structure," *Midland Corporate Finance Journal* 1, 6-16; also in Stern, J. M. and D. H. Chew Jr. (ed.), 1986, *The Revolution in Corporate Finance,* pp. 91-99, Oxford: Basil Blackwell.

Ross, Sheldon, 1993, *Introduction to Probability Models*, Academic Press, New York.

Ross, Stephen, Randolph Westerfield and Jeffrey Jaffe, 2010, *Corporate Finance*, McGraw-Hill, New York.

Shreve, Steven, 2004a, *Stochastic Calculus for Finance I: The Binominal Asset Pricing Model*, Springer-Verlag, New York.

Shreve, Steven, 2004b, *Stochastic Calculus for Finance II: Continuous-Time Models*, Springer-Verlag, New York.

Wilmott, Paul, 2007, *Paul Wilmott Introduces Quantitative Finance*, John Wiley & Sons, West Sussex, England.